Stability of striped phases in two-dimensional dipolar systems

1994 J. Phys.: Condens. Matter 6 L731

(http://iopscience.iop.org/0953-8984/6/47/002)

J. Phys.: Condens. Matter 6 (1994) L731–L734. Printed in the UK

LETTER TO THE EDITOR

Stability of striped phases in two-dimensional dipolar systems

J P Whitehead† and K De'Bell‡

† Department of Physics, Memorial University of Newfoundland, St John's, Newfoundland, A1B 3X7, Canada
‡ Department of Physics, Trent University, Peterborough, Ontario, K9J 7B8, Canada

Received 16 September 1994

Abstract. The energy of striped phases in two-dimensional dipolar Ising systems is calculated in the asymptotic limit of wide stripes. Within each stripe the dipole moments are mutually parallel, and antiparallel to dipole moments in the adjacent stripes. The possibility of observing such phases in physical systems is briefly discussed.

Two-dimensional spin systems in which the spin is constrained to align perpendicularly to the plane are of interest as models of a variety of systems. In particular, this model may be applicable to magnetism in metal on metal films [1] and in rare earth layered systems [2]. While the dipolar interaction in such systems may be small relative to the exchange interaction, the long-range character of the dipolar interaction means that it plays a fundamental role in determining the magnetic properties. Recently, two independent studies [3, 4] have noted that striped phases will occur in lattice Ising models, if the exchange interaction favours ferromagnetic ordering of the magnetic moments. Earlier work on continuum models, by Garel and Doniach [5] and Yafet and Gyorgy [6], had also noted the possibility of striped phases forming in magnetic films as a consequence of the dipolar interaction, while Czech and Villain [7] have discussed the possibility of a 'chequerboard phase'.

In [3] and [4] the ground state energies of the striped phases were calculated numerically. These calculations were limited to stripes with maximum widths of a few hundred lattice spacings. Consequently, these studies did not determine if there exists a critical value of the exchange interaction parameter above which the ferromagnetic state is stable with respect to formation of a striped phase with stripes of large but finite width. In figure 2 of [3], a ferromagnetic region is indicated above a critical value of the exchange parameter, although the text notes that this question is still open.

In a recent paper [8] we obtained an analytic expression for the dipolar energy associated with the striped phase of a two-dimensional dipolar Ising system as a function of the width of the stripe h. The analysis is based on the assumption that the width of the stripe is large and neglects terms of order h^{-2} and higher. A comparison of the analytic result of [8] and the numerical results of [3] and [4] shows that the agreement is excellent even for relatively small values of h ($\simeq 8$). In addition to providing a relatively accurate approximation to the dipolar energy over a wide range of h the result proves that the dipolar interaction will always destabilize the ferromagnetic ground state of the ferromagnetic Ising model.

0953-8984/94/470731+04$19.50

In this letter, we summarize the result presented in [8] and establish the correspondence with the earlier work. Finally we compare the theoretical predictions with some data on metal on metal films.

The starting point of the analysis in [8] is the expression for the dipolar energy, which may be written in dimensionless form as

$$E_{\text{dip}}(\{\sigma_i\}) = \sum_{i \neq j} \frac{\sigma_i \sigma_j}{|\boldsymbol{R}_{ij}|^3} \tag{1}$$

where $\sigma_i = \pm 1$ denotes the spin variable associated with the ith lattice site and $\boldsymbol{R}_{ij}$ denotes the distance between the lattice sites i and j. For the striped phase consisting of domains in which the spins are aligned in parallel to form stripes consisting of h columns, with the direction alternating between successive stripes, this may be written as

$$E_{\text{dip}}(\{\sigma_i\}) = \frac{2N^2}{h^2} \left(\sum_{n=0}^{(h-1)/2} \frac{\Gamma(K_n)}{\sin^2(K_n/2)} \right) \tag{2}$$

where N denotes the number of sites on the lattice, $\{K_n = \frac{2\pi}{h}(n + \frac{1}{2})\}$ denotes the set of reciprocal lattice vectors associated with the magnetic superlattice and $\Gamma(\boldsymbol{Q})$ is defined as

$$\Gamma(\boldsymbol{Q}) = \sum_{\boldsymbol{R} \neq 0} \frac{e^{i\boldsymbol{Q}\cdot\boldsymbol{R}}}{|\boldsymbol{R}|^3} \,. \tag{3}$$

The long-range character of the dipolar interaction manifests itself in the long-wavelength behaviour of $\Gamma(\boldsymbol{Q})$ giving rise to a non-analytic term proportional to Q [9]

$$\Gamma(\boldsymbol{Q}) = \Gamma_0 - 2\pi Q + \tilde{\Gamma}(\boldsymbol{Q}) \,. \tag{4}$$

The presence of the linear term in $\Gamma(\boldsymbol{Q})$ leads to a logarithmic contribution in the leading asymptotic correction to the dipolar energy of the striped phase, given by

$$\lim_{h\to\infty} E_{\text{dip}}(h) = E^0_{\text{dip}} - \frac{1}{h}(A + B\log h) + \mathcal{O}\left(\frac{1}{h^2}\right) \tag{5}$$

where E^0_{dip} denotes the dipolar energy of the ferromagnetic ground state. The parameter B may be calculated analytically to give $B = 8$, while A may be evaluated numerically to yield $A = 9.167$.

Combining the above expression with the exchange energy arising from a ferromagnetic nearest-neighbour interaction yields the following form for the total energy of the striped phase

$$E = E_{\text{F}} - \frac{1}{h}(A - 2J + B\log h) + \mathcal{O}\left(\frac{1}{h^2}\right) \tag{6}$$

where $E_{\text{F}} \equiv E^0_{\text{dip}} - 2J$ denotes the ground state energy of the ferromagnetic state. Minimizing the total energy with respect to h we obtain an expression for the equilibrium thickness, which we denote by h^*, as

$$h^* = h_0 \exp\frac{2J}{B} \tag{7}$$

where $h_0 = \exp(1 - A/B) \simeq 1$. Substituting this into (6) we obtain the following expression for the ground state energy

$$E = E_{\text{F}} - \frac{B}{h^*} + \mathcal{O}\left(\frac{1}{h^{*2}}\right) \tag{8}$$

from which we see that the ground state energy of the striped phase is *always* less than that of the corresponding ferromagnetic phase. Consequently the striped phase is always stable with respect to the ferromagnetic phase no matter how large the exchange interaction.

Qualitatively our conclusions agree with the earlier work of Garel and Doniach [5] and Yafet and Gyorgy [6]. However, in contrast to the derivation given in [8], both these works are based on a continuum description of the magnetic superlattice and assume that the spatial variation of the magnetic moment is small on the scale of the lattice constant a_0. In order to understand the correspondence between these approaches and our derivation we note that the leading order correction to the dipolar energy ($\simeq h^{-1}\log h$) arises as a consequence of a contribution to the summation in (2) of the form $\sum_n K_n^{-1}$. Since K_n is constrained to lie in the first Brillioun zone (i.e. $-\pi \geqslant K_n \geqslant \pi$), this contribution is finite, varying as $\log h$ in the limit of large h.

The expression for dipolar energy given by Yafet and Gyorgy [6] contains a similar contribution ($\simeq \sum_n K_n^{-1}$) [10]. However, since they have used a continuum approximation in their analysis, the summation is not restricted to the first Brillioun zone. Instead the cut-off arises as a consequence of the modification to the Fourier amplitudes (the coefficients b_m in [6]) at large wavenumber due to the finite width of the domain wall. Thus they find that the dipolar energy contains a contribution of the form $\log w/\Lambda$, where w and Λ denote the width of the domain wall and the modulation length respectively. A similar contribution to the dipolar energy was obtained earlier by Garel and Doniach [5].

The apparent correspondence between between the dipolar energy given by Yafet and Gyorgy [6] and the derivation in [8] that one obtains by setting $w = a_0$ in the former (i.e. assuming that the domain wall in [6] is one lattice constant wide) is, however, somewhat misleading. A more useful way to understand the relationship between the two results is to consider the two results as the limiting cases of the more general expression that includes the underlying lattice (as in [8]) and the finite width of the domain wall (as in [6]). The close correspondence between the functional forms of the dipolar energy given in references [6] and [8] suggests that the general case may be described in terms of certain crossover functions involving only the lattice spacing and the domain wall width.

These results raise the question of the observation of striped phases in physical systems. In the rare earth layered systems such as the high-T_c superconductors and related compounds, only simple antiferromagnetic ordering of the out-of-plane magnetic moments is observed. This implies that the exchange interaction is negative and Monte Carlo studies [11] have confirmed that a consistent picture is obtained if such an interaction is assumed. On the other hand macroscopic domains are observed in metal on metal overlayers and have been interpreted as realizations of stripes of the type discussed by Yafet and Gyorgy [12].

If the strength of the exchange interaction is assumed to be large compared with the dipolar interaction, the critical temperature is given to a good approximation by the Onsager solution

$$k_B T_c = 2.269\left(J\mu_{\rm eff}^2/a^3\right) \tag{9}$$

where $\mu_{\rm eff}$ is the effective magnetic moment and a is the lattice spacing. Substituting this into the previous expression for h^* we obtain

$$h^* \simeq \exp\left(\frac{a^3 T_c}{5.65\mu_{\rm eff}^2}\right) \tag{10}$$

where, in this last expression, we have units in which a is measured in ångströms and $\mu_{\rm eff}$ is measured in Bohr magnetons. Typical values of the parameters for transition metal films based on experimental studies of Au on Cu [13, 14], are $a = 2.85$ Å, $\mu_{\rm eff} \simeq 3\mu_B$ and

$T_c \simeq 450$ K. (The approximate value of T_c is inferred from the hysteresis loops for a film of 4.5 monolayers shown in [14].) The h^* corresponding to these values is far larger than that for any experimental sample. Therefore we would not expect to observe the striped phase ground states in such films. However the value of h^* is very sensitive to the $\mu_{\rm eff}$ of the system. For a rare earth such as Dy, typical values are $\mu_{\rm eff} \simeq 10\mu_{\rm B}$ with $a \simeq 3$ Å. Consequently macroscopic stripes of, say, 10 000 lattice spacings would occur in systems with a critical temperature of

$$T_c \simeq 190 \text{ K.} \tag{11}$$

It should however be noted that the spatial anisotropy of the magnetic moments in thin films is not in general infinite in the direction perpendicular to the plane as in the Ising model used in references [3] and [4]. Moreover domains observed in thin films do not have the rigid vertical walls of the stripes in the ground state calculations. Therefore there will be a contribution from an entropic term associated with the wall fluctuations. However both of these considerations are likely to favour domain formation. It would be interesting to examine in more detail the effect of thermal fluctuations using simulations on the dipolar Ising model. Given the relationship between the continuum models and the Ising model discussed in this paper, such studies would complement the recent work of Kashuba and Pokrovsky [15] who have examined the effects of fluctuations within a continuum field model.

The authors thank A B McIsaac for relevant discussions. This work is supported in part by the Natural Sciences and Engineering Research Council of Canada.

References

[1] Pescia D and Pokrovsky V L 1990 *Phys. Rev. Lett.* **65** 20
[2] De'Bell K and Whitehead J P 1991 *J. Phys.: Condens. Matter* **3** 2431
[3] Taylor M B and Gyorffy B L 1993 *J. Phys.: Condens. Matter* **5** 4527
[4] McIsaac A B 1992 Monte Carlo study of the two-dimensional dipolar Ising system *MSc Thesis* Memorial University of Newfoundland
[5] Garel T and Doniach S 1982 *Phys. Rev.* B **26** 325
[6] Yafet Y and Gyorgy E M 1988 *Phys. Rev.* B **38** 9145
[7] Czech R and Villain J 1989 *J. Phys.: Condens. Matter* **1** 619
[8] McIsaac A B, Whitehead J P and De'Bell K 1994 *Preprint*
[9] Maleev S V 1976 *Zh. Eksp. Teor. Fiz.* **70** 2374 (*Sov. Phys.–JETP* **43** 1240)
[10] In equation (14) of [6], the summation over the index m corresponds to a summation over the reciprocal lattice vectors of the magnetic superlattice. For small wavenumber, $m \ll \delta$, the coefficients $b_m \simeq m^{-1}$, which implies that the terms in the second summand vary as m^{-1}, while for large wavenumber, $m > \delta$, the coefficients $b_m \simeq m^{-3} \cos m\delta$. Hence we see that the parameter δ acts as an effective cut-off giving rise to the $\log \delta$ dependence of the dipolar energy.
[11] McIsaac A B, Whitehead J P, De'Bell K and Narayanan K S S 1992 *Phys. Rev.* B **46** 6387
[12] Allenspach R, Stampanoni M and Bischof A 1990 *Phys. Rev. Lett.* **65** 3344
[13] Guo G Y, Temmerman W M and Ebert H 1991 *J. Phys.: Condens. Matter* **3** 8205
[14] Liu C, Moog E R and Bader S D 1988 *Phys. Rev. Lett.* **60** 2422
[15] Kashuba A and Pokrovsky V L 1993 *Phys. Rev. Lett.* **70** 3155